Tutorial Note for Math2012E

May 18, 2016

1 Geometric Interpretations of Equations

- distance between two points

$$d(P,Q) = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

- Equation of Sphere

$$Standard : (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = R^2$$
$$General : x^2 + y^2 + z^2 + Dx + Ey + Fz + G = 0$$

- Equation of Open Ball

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 < R^2$$

- Equation of Closed Ball

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \leq R^2$$

- Equation of Open Annular

$$r^2 \leq (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 < R^2$$

- Equation of Plane

$$Ax + By + Cz = D$$

- Special Examples

$$x = a, y = b, z = c$$
$$x + y = a, x + z = b, y + z = c$$

- Combination of two equations

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$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = R^2, y = a < R$$

2 Problems of Chapter 12.1

cf. Chapter 12.1 Theory and Examples
Two way to solve the problem relevant to geometry:

- Translate into equation or formula
- Geometric interpretation

For detail, please come to tutorial class.

3 More Definitions in Chapter 12.2

- unit vector: $\frac{\vec{v}}{|\vec{v}|}$ is the unit vector in the direction of $\vec{v}$
- middle point of P,Q :

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

4 More Definitions in Chapter 12.3

- Vector $\vec{u}, \vec{v}$ are orthogonal(perpendicular) if and only if $\vec{u} \cdot \vec{v} = 0$
- The vector projection of $\vec{u}$ onto a nonzero vector $\vec{v}$ is

$$proj_v \vec{u} = |\vec{u}| \cos\theta \frac{\vec{v}}{|\vec{v}|} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \vec{v}$$

- the scalar component of $\vec{u}$ in the direction of $\vec{v}$ is the scalar

$$|\vec{u}| \cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} = \vec{u} \cdot \frac{\vec{v}}{|\vec{v}|}$$

- The work done by a constant force $\vec{F}$ acting through a displacement $\vec{D}$ is

$$W = \vec{F} \cdot \vec{D}$$

- Cauthy-Schwartz inequality

$$|\vec{u} \cdot \vec{v}| \leq |\vec{u}||\vec{v}|$$

$$(x_1x_2 + y_1y_2 + z_1z_2)^2 \leq (x_1^2 + y_1^2 + z_1^2)(x_2^2 + y_2^2 + z_2^2)$$

- dot product doesn't satisfy cancellation law

5 More Definitions in Chapter 12.4

- $|\vec{u} \times \vec{v}|$ is the area of the parallelogram
- Torque
  Force $\vec{F}$, level arm $\vec{r}$
  Direction of torque vector: the axis of bolt determined by right-hand law
  Magnitude of torque vector $= |\vec{r}||\vec{F}| \sin\theta$
  or equivalently, let $\vec{T}$ be Torque vector, we have

$$\vec{T} = \vec{r} \times \vec{F}$$

- Three vectors $\vec{u}, \vec{v}, \vec{w}$ lie in a plane if and only if

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$$

- Cross product doesn't satisfy cancellation law
- Double cancellation law is satisfied, i.e.

$$\vec{u} \times \vec{v} = \vec{u} \times \vec{w}, \vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w} \Rightarrow \vec{v} = \vec{w}$$

6 More Definitions in Chapter 12.5

No more definitions